The Origins of Science and the Geometrical Roots of the Triple Tau
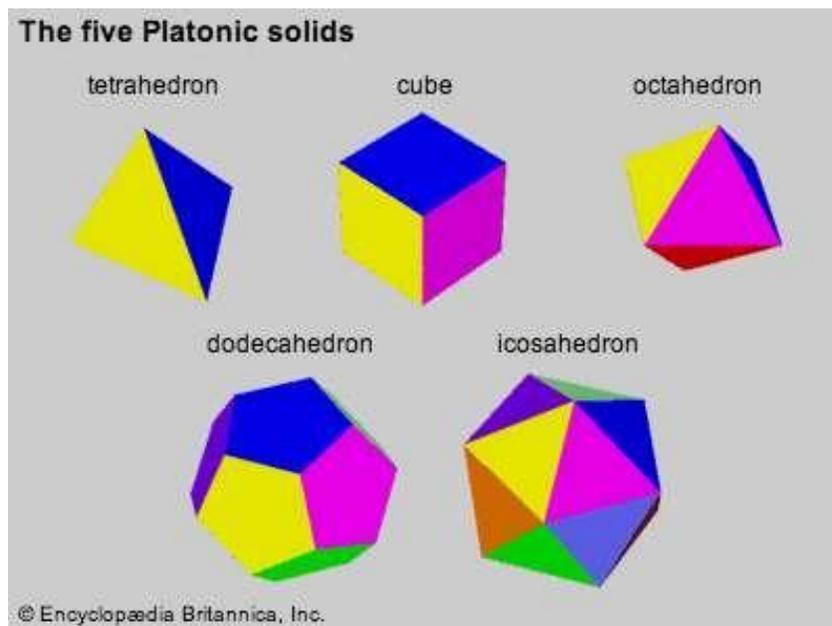
For 2000 years, from the time of Plato (428 – 384 BC) to the Renaissance, the prevailing belief was that there were just five elements. The common elements were earth, air, fire and water. Everything on earth, including plants animals and humans, were made up of these elements. The special fifth element was believed to have been used by God to create the universe. Each of these five elements was associated with one of the five regular three dimensional geometric shapes known as the platonic solids.

Although these beliefs seem strange to us from the vantage point of the 21st century, when we know that there are more than 100 elements, the geometry of the platonic solids has continued to inspire artists and scientists to this very day. For the Operative Masons a study of these objects would have been a necessary part of their further education. The Platonic bodies extended their consideration from the two dimensional planar geometry (squares, circles and triangles) that characterizes the three earlier degrees into three dimensional Royal Arch Mason architecture.

The ancient Greek credited with originating this theory of the elements was Empedocles (430 – 409 B.C.). He and Plato believed that mathematics would reveal the structure of the world. The smallest fractions of the elements had geometric structure; **Fire** was **the tetrahedron**, **Air, the octahedron**, **Earth, the cube** and **Water, the icosahedrons. The mysterious fifth element was reserved for God, the pentagonal Dodecahedron.**

Note that the four common elements also logically represent the four phases of matter: plasma, gas, liquid and solid. Aristotle had a slightly different notion that the world was made up of properties of hot and cold, dry and wet; Hot and dry, Tetrahedron, Hot and wet, Octahedron, Cold and wet, Icosahedron and Cold and dry, Cube. The pentagonal dodecahedron did not figure in Aristotle's system.

This chart will demonstrate the models to you. I am sure they are not new to you, but I am hoping that you will perhaps enjoy a fresh look at them and we will review the simple proof as to why there are only five regular bodies that can exist at least in this dimension!

The five Platonic solids

tetrahedron   cube   octahedron

dodecahedron   icosahedron

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The first Platonic body is the tetrahedron. It has four equilateral triangular faces. All edges are the same length and all four vertices are identical. **Three equilateral triangles join at every vertex**.

The second is the octahedron. It has eight equilateral triangular faces. All edges are the same length and all six vertices are identical. F**our equilateral triangles join at every vertex.**

The third is the icosahedron. It has 20 regular triangular faces. All edges are the same length and all 12 vertices are identical. F**ive equilateral triangles join at every vertex.**

The fourth platonic body is the cube. It has six square faces. All edges are the same length and all eight vertices are identical. **Three squares join at every corner or vertex.**

The fifth and final platonic solid is the pentagonal dodecahedron. It has 12 faces each a pentagon (five sides). All the edges are the same length and all 20 vertices are identical. **Three pentagons join at every vertex.** So here are the five Platonic Bodies: The tetrahedron, the octahedron, the icosahedron, the cube and the pentagonal dodecahedron.

It is very easily proved that these are the only regular geometric solids that can exist. There are five and no more. Looking at the vertices of these solids provides the key to the mathematical proof.

The tetrahedron, octahedron and the icosahedron

These are the only three regular geometric shapes that can be made from equilateral triangles. The vertices have three, four and five equilateral triangles. If you try to join six equilateral triangles you end up with a two dimensional hexagon.

The Cube

This is the only regular geometric shape that can be made from squares. If you try to add another square, the result is a two dimensional square.

The Pentagonal dodecahedron

This is the only regular geometric shape that can be made from pentagons alone. If you try to join four pentagons they simply overlap each other prohibiting the formation of any structure.

It must further be stated that no regular geometric solids can be formed with faces that have more than five sides. So there you have the simple proof that there are only five regular geometric solids.

There are of course other three dimensional geometric bodies, possibly thousands of them, but none of them have the peculiar property of having all vertices identical AND all faces the same shape.

The Cuboctahedron (a mixture of equilateral triangles and squares) has all vertices identical but clearly all faces are not the same shape.

The Rhombic Dodecahedron (composed of all diamond shaped faces) clearly has all faces identical, but not all vertices are the same.

The five platonic solids also have another remarkable property. Each of the vertices is equidistant from the geometric centre of each body. This means that if any of the platonic bodies were placed inside a sphere all the vertices touch the inside surface of that sphere.This interesting property led some early scientists, notably Kepler, to propose a planetary model where the orbits of the planets in our solar system were described by platonic bodies nested one inside the other. The ink was barely dry on this theory, however, before other more plausible theories that fit the observed data more closely replaced it.

Geometricians have been aware of these five special objects and their properties for 2,500 years. Plato (427 - 348 B.C.), the famous Greek philosopher, describes these solids in his book "Timaeus", hence, the association of his name with these structures. Hippocrates (460 – 377 B.C.), the Greek father of medicine developed a four-humour theory. He believed that moods and emotions in humans was caused by imbalances of blood, bile and phlegm. I have coloured the Platonic bodies to reflect the medical theory of Hippocrates; Tetrahedron – red for blood, Octahedron - yellow for yellow bile, Cube - black for black bile, and Icosahedron - white for phlegm.**The fifth platonic body, the pentagonal dodecahedron, was presumably too powerful to be associated with the human body and was not mentioned by Hippocrates in his humour theory. It would not be too great a logical leap to say that the pentagonal dodecahedron is very likely the original source of the pentagonal star as the Masonic symbol for God.**

So, that brings our attention back to Masonry and its connection to the platonic solids. There is a geometric link between the five platonic bodies and the triple tau. I think the best way to show this link is for us to calculate the external degrees of each of the platonic solids reduced to right angles and see where that leads us.

Let's start with the tetrahedron. It has four equilateral triangles, the same number as the triangles formed by the greater and lesser lights of Masonry. It is a basic law of geometry that the sum of all the interior angles of a triangle equal 180 degrees or two right angles. Therefore the tetrahedron, which is made up of four triangles, has a total number of external degrees equivalent to 8 right angles.

The octahedron has eight triangles (twice as many triangles as the tetrahedron) and therefore has the equivalent of 16 right angles. The icosahedron has 20 triangles (five times as many as the tetrahedron). The icosahedron therefore has 40 right angles. The cube has six faces each with four right angles which means the external faces on the cube have 24 right angles. The pentagonal dodecahedron is a little more tricky. Each pentagonal face has five angles of 108 degrees. 108 times five = 540 degrees = 6 right angles. So each face has the equivalent degrees of 6 right angles. The pentagonal dodecahedron has 12 faces so the external degrees for this body are equivalent to 72 right angles.

So here are the platonic bodies represented in terms of right angles; Tetrahedron, eight right angles; Octahedron, 16 right angles; Cube, 24 right angles; Icosahedron, 40 right angles and the Pentagonal dodecahedron, 72 right angles.

What is the connection between these numbers?

They are all divisible by eight. How many right angles are in the triple tau? The Chapter's book of work (on page 130) says it this way: "This symbolical arrangement (referring to the greater and lesser lights) corresponds with the mysterious triple Tau which forms two right angles on each of the exterior lines and two others by their union in the centre." The triple tau therefore has eight right angles.

We can now represent all the platonic bodies by a corresponding number of triple taus. The tetrahedron is thus equivalent to one triple tau; The octahedron to two triple taus; The cube to three triple taus; The icosahedron to five triple taus; and The pentagonal dodecahedron to nine triple taus.

So in Summary perhaps I can add a commentary. I believe these geometric objects have an intrinsic beauty all their own and men through the ages have questioned their creation and their significance to the world around them. As a result the Platonic Solids have helped raise ignorance from despair and establish happiness in the paths of science.

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References
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